Drexel University, Department of Materials Science and Engineering

Materials Science and Engineering II – Winter 2008

Homework 1

H1.Q1: Calculate the dilatation Δ in the uniaxial elastic extension of a bar of material — assuming strains are small — in terms of ν and the tensile strain, ε. Hence find the value of Δ for which the volume change during elastic deformation is zero.

For small strains, the volume change per unit volume is $\delta V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$. In uniaxial tension $\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$, thus with tensile strain $\varepsilon_1 \equiv \varepsilon$ the expression becomes $\delta V = \varepsilon - \nu\varepsilon - \nu\varepsilon = (1-2\nu)\varepsilon$.

Hence $\Delta = \frac{\delta V}{V} = (1-2\nu)\varepsilon$ and Δ is zero when $\nu = 0.5$ (the value for rubber, for example).

H1.Q2: Poisson's ratio for most metals is about 0.3. For rubber it is close to 0.5. What are approximate volume changes in these materials during an elastic tensile strain of ε.

Using the expression derived above, $\Delta = \frac{\delta V}{V} = (1-2\nu)\varepsilon$, we find that for most metals $\Delta \approx 0.4\varepsilon$ and that for rubber $\Delta = 0$.

H1.Q3: What would be the ideal Poisson's ratio for the cork in a bottle of wine. Would a rubber bung be equally suitable?

Because it should be as easy to position the cork in the bottle neck as to remove it, the ideal Poison's ratio would ensure that the material expands neither under compression nor under tension. A material will behave that way if it has $\nu = 0$.

H1.Q4: The cable of a hoist has a cross-section of 80 mm^2. The hoist is used to lift a crate weighing 500 kg. What is the stress in the cable? The free length of the cable is 3 m. How much will it extend if it is made of steel (Young's modulus 200 GPa)? How much if it is made of polypropylene, PP (Young's modulus 1.2 GPa)?

The stress in the cable is $\sigma = \frac{F}{A} = \frac{500\text{ kg} \times 9.81\text{ N/kg}}{80\text{ mm}^2} = 61.3\frac{\text{N}}{\text{mm}^2} = 61.3\text{ MPa}$. The strain in the cable is $\varepsilon_{steel} = \frac{\sigma}{E} = \frac{61.3\text{ MPa}}{200\text{ GPa}} = 0.307\times10^{-3}$ and $\varepsilon_{PP} = \frac{\sigma}{E} = \frac{61.3\text{ MPa}}{1.2\text{ GPa}} = 0.051$. The cable will therefore extend by 0.92mm in the case of steel and by 153mm in the case of PP.

H1.Q5: The rails of a railroad track are welded together at their ends (to form continuous rails and thus eliminate the cladding sounds of the wheels) when the temperature is 20ºC. What compressive stress σ is produced in the rails when they are heated by the sun to 50ºC if the coefficient of thermal expansion is $\alpha = 12 \times 10^{-6}$/ºC and the Young's modulus is $E = 200$ GPa?

The thermal strain generated in the rails is $\varepsilon = \alpha(\Delta T)$. The stress resulting from this strain is

$$\sigma = E\alpha(\Delta T) = 200\text{ GPa} \times \frac{12\times10^{-6}}{°\text{C}} \times 30°\text{C} = 72\text{ MPa}.$$